#### **On Simulation and Optimization of Freeway Network Operations**

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## **Progress from last PSG meeting**

- 1 Freeway traffic control via Ramp Metering and Variable Speed Limit using a mesoscopic model
- 2. Control of ramp metering based on reinforcement learning

# Freeway Traffic Control via Ramp Metering and Variable Speed Limit using a mesoscopic model

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#### Background



Average free flow speeds

https://www.mainroads.wa.gov.au/about-main-roads/news-media/smart-freeway-technology-upgrades/

# Traffic monitoring and Control systems

- Traffic congestion has a significant impact on economic activity throughout much of the world.
- An essential step towards active congestion control is the creation of accurate, reliable traffic monitoring and control systems.
- These systems usually run algorithms which rely on mathematical models of traffic used to power estimation and control schemes.

#### **Road Network**



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#### The DE-CTM optimisation model:

$$TTS = \min\left[\sum_{t=\tau}^{T}\sum_{i\in C}TL_{i}k_{i}(\alpha,\beta,t)\right]$$

 $0 \le k_i(t) \le k_{jam}$ 

 $\alpha \in [\alpha_{min}, \alpha_{max}]$ : probability from upstream normal cell;

 $1 - \alpha$ : probability from upstream merge cell

الليل

upstream merge cell

$$\begin{aligned} \frac{n_i \partial k_i}{\partial t} + \frac{\partial (n_i q_i)}{\partial x} &= g(\alpha, \beta, t) \\ r_{i,j}(t) &= \min \left[ (1 - \alpha)q_{i-1}(t); \quad Q_r; \quad m_i(t-1) + \frac{T}{L_j} (d_{i,j} + r_{i,j}(t-1)) \right], \quad j = 1, \dots, 8 \\ \vdots \quad \sum_{t=\tau}^{\tau+H_c-1} T \left\{ \sum_{i \in C_M} r_{i,j}(t) + n_i(q_i(t) - Q_i) \right\} &\leq 0, \qquad j = 1, \dots, 8 \end{aligned}$$

Constraints:

# Some results $\alpha_m \in [-0.25, 0.25], \beta_{VSL} \in [0.25, 1.0]$

Let  $q_{max} = 1800$  vph on freeway  $q_{max} = 1300$  vph on on/off-ramp

Case study		Segment	Free speed (km/hr)	Capacity(veh/hr)
I	а	All segments	100	1800
	b	All segments	70	1800
Ш	а	17	100	450
	b	26	100	450
Ш	а	17	70	450
	b	26	70	450



# Case I(b): Heatmap plot of density and flow rate

#### Free speed 100

Traffic density (veh/km)

Traffic flow rate (veh/hr)



## Case I(a): RM control and Variable speed limit

#### Ramp Metering at five on-ramps from Farrington to Cranford



Variable Speed Limits from Cranford to Mill Points



## Case I (a): Heatmap plot of density and flow rate Free speed 70

Traffic density (veh/km)

Traffic flow rate (veh/hr)



### Case I (b): RM control and Variable speed limit

#### Ramp Metering at five on-ramps from Farrington to Cranford



Variable Speed Limits from Cranford to Mill Points



# Case II(a): Heatmap plot of density and flow rate Segment 17

#### Traffic density (veh/km)

#### Traffic flow rate (veh/hr)



## Case II(a): RM control and Variable speed limit

#### Ramp Metering at five on-ramps from Farrington to Cranford



#### Variable Speed Limits from Cranford to Mill Points



# Case II(b): Heatmap plot of density and flow rate Segment 26

600

500

400

300

200

100

Density (veh/km) on Freeway with 25% flow capacity at the 26th segment A0.32.0.C3 A0.29.0.C2 A0.27.0.C26 A0.27.0.C16 A0.27.0.C6 Bridge A0.24.0.C1 A0.20.0.C5 A0.18.0.C13 ž A0.18.0.C3 Hay to A0.17.0.C0 Soe A0.12.0.C0 A0.9.0.C3 A0.7.0.C1 A0.4.0.C4 A0.2.0.C0 A0.0.0.C0 6:00:00 6:15:00 6:30:00 6:45:00 7:00:00 7:15:00 7:45:00 8:00:00 8:15:00 8:30:00 8:45:00

Traffic density (veh/km)

#### Traffic flow rate (veh/hr)



### Case II(b): RM control and Variable speed limit



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Variable Speed Limits from Cranford to Mill Points



## Case III(a): Heatmap plot of density and flow rate Segment 17

Traffic density (veh/km)

Traffic flow rate (veh/hr)





## Case III(a): RM control and Variable speed limit



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# Case III(b): Heatmap plot of density and flow rate

#### Segment 26

Traffic density (veh/km)

Traffic flow rate (veh/hr)



## Case III(b): RM control and Variable speed limit

#### Ramp Metering at five on-ramps from Farrington to Cranford



#### Variable Speed Limits from Cranford to Mill Points



## 2. Control of ramp metering based on reinforcement learning

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## Control of ramp metering based on reinforcement learning



- state: queue length, mean waiting time, mean speed....
- action: switching signal phase,  $a \in \{0,1\}$
- policy:  $\pi(a \mid s)$  $\pi(1 \mid s) = 0.8$  $\pi(0 \mid s) = 0.2$



#### reward: mean waiting time, mean time, total time spent....

state transition: old state  $\rightarrow$  new state  $S' \sim p(\cdot | s, a)$ 

• (state, action, reward) trajectory:

$$S_1, a_1, r_1, S_2, a_2, r_2, \cdots, S_n, a_n, r_n.$$

• One episode is from the the beginning to the end



## **Randomness in Returns**

**Definition:** Discounted return (at time *t*).

•  $U_t = R_t + \gamma R_{t+1} + \gamma^2 R_{t+2} + \cdots + \gamma^{n-t} R_n$ .

At time t, the rewards,  $R_t$ ,  $\cdots$ ,  $R_n$ , are random, so the return  $U_t$  is random.

- Reward  $R_i$  depends on  $S_i$  and  $A_i$ .
- States can be random:  $S_i \sim p(\cdot | s_{i-1}, a_{i-1})$ .
- Actions can be random:  $A_i \sim \pi(\cdot | s_i)$ .
- If either  $S_i$  or  $A_i$  is random, then  $R_i$  is random.

# Action-Value Function $Q_{\pi}(s, a)$

**Definition:** Discounted return.

• 
$$U_t = R_t + \gamma R_{t+1} + \gamma^2 R_{t+2} + \cdots + \gamma^{n-t} R_n$$
.

**Definition:** Action-value function.

• 
$$Q_{\pi}(s_t, \mathbf{a}_t) = \mathbb{E} \left[ U_t \mid S_t = s_t, A_t = a_t \right].$$

- $Q_{\pi}(s_t, a_t)$  depends on  $s_t, a_t, \pi$ , and p.
- $Q_{\pi}(s_t, \mathbf{a}_t)$  is dependent of  $S_{t+1}, \dots, S_n$  and  $A_{t+1}, \dots, A_n$ .

## State-Value Function $V_{\pi}(s)$

**Definition:** Discounted return.

• 
$$U_t = R_t + \gamma R_{t+1} + \gamma^2 R_{t+2} + \cdots + \gamma^{n-t} R_n.$$

**Definition:** Action-value function.

• 
$$Q_{\pi}(s_t, \mathbf{a}_t) = \mathbb{E} [U_t | S_t = s_t, A_t = \mathbf{a}_t].$$

**Definition:** State-value function.

• 
$$V_{\pi}(s_t) = \mathbb{E}_{A}[Q_{\pi}(s_t, A)]$$

# Action-Value Functions *Q*(*s*, *a*)

**Definition:** Discounted return (aka cumulative discounted future reward).

• 
$$U_t = R_t + \gamma R_{t+1} + \gamma^2 R_{t+2} + \gamma^3 R_{t+3} + \cdots$$

**Definition:** Action-value function for policy  $\pi$ .

• 
$$Q_{\pi}(s_t, \boldsymbol{a}_t) = \mathbb{E}\left[U_t | S_t = s_t, \boldsymbol{A}_t = \boldsymbol{a}_t\right].$$

**Definition:** Optimal action-value function.

• 
$$Q^{\star}(s_t, \boldsymbol{a_t}) = \max_{\pi} Q_{\pi}(s_t, \boldsymbol{a_t}).$$

• Whatever policy function  $\pi$  is used, the result of taking  $a_t$  at state  $s_t$  cannot be better than  $Q^*(s_t, a_t)$ .

# Approximate the Q Function

**Goal:** Win the game ( $\approx$  maximize the total reward.)

**Question:** If we know  $Q^*(s, a)$ , what is the best action?

• Obviously, the best action is  $a^* = \operatorname{argmax} Q^*(s, a)$ .

**Challenge:** We do not know  $Q^*(s, a)$ .

- Solution: Deep Q Network (DQN)
- Use neural network  $Q(s, \mathbf{a}; \mathbf{w})$  to approximate  $Q^*(s, \mathbf{a})$ .



# How to apply TD learning to DQN?

**Identity:**  $U_t = \mathbf{R}_t + \mathbf{\gamma} \cdot U_{t+1}$ .

#### **TD learning for DQN:**

- DQN's output,  $Q(s_t, a_t; \mathbf{w})$ , is an estimate of  $U_t$ .
- DQN's output,  $Q(s_{t+1}, a_{t+1}; \mathbf{w})$ , is an estimate of  $U_{t+1}$ .

• Thus, 
$$Q(s_t, a_t; \mathbf{w}) \approx r_t + \gamma \cdot Q(s_{t+1}, a_{t+1}; \mathbf{w})$$
.  
Prediction TD target

# **Train DQN using TD learning**

- Prediction:  $Q(s_t, a_t; \mathbf{w}_t)$ .
- TD target:

$$y_t = r_t + \gamma \cdot Q(s_{t+1}, a_{t+1}; \mathbf{w}_t)$$
$$= r_t + \gamma \cdot \max_a Q(s_{t+1}, a; \mathbf{w}_t).$$

• Loss:  $L_t = \frac{1}{2} [Q(s_t, a_t; \mathbf{w}) - y_t]^2$ .

• Gradient descent: 
$$\mathbf{w}_{t+1} = \mathbf{w}_t - \alpha \cdot \frac{\partial L_t}{\partial \mathbf{w}} |_{\mathbf{w} = \mathbf{w}_t}$$
.

# **Temporal Difference (TD) Learning**

Algorithm: One iteration of TD learning.

- 1. Observe state  $S_t = s_t$  and perform action  $A_t = a_t$ .
- 2. Predict the value:  $q_t = Q(s_t, a_t; \mathbf{w}_t)$ .
- 3. Differentiate the value network:  $\mathbf{d}_t = \frac{\partial Q(s_t, a_t; \mathbf{w})}{\partial \mathbf{w}} |_{\mathbf{w} = \mathbf{w}_t}$ .
- 4. Environment provides new state  $s_{t+1}$  and reward  $r_t$ .
- 5. Compute TD target:  $y_t = r_t + \gamma \cdot \max_a Q(s_{t+1}, a; \mathbf{w}_t)$ .
- 6. Gradient descent:  $\mathbf{w}_{t+1} = \mathbf{w}_t \alpha \cdot (\mathbf{q}_t \mathbf{y}_t) \cdot \mathbf{d}_t$ .



 $W_{ijt}$ : the waiting time of vehicle i on ramp j for time t

T = 7200, J = 7



# Result:



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# THANK YOU

# **For Your Attention**

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